

# Numerical Algorithms and Observer Design for Fractional Order Systems

Yiming Chen<sup>1,2,3</sup>, Driss Boutat<sup>3</sup>, Dayan Liu<sup>2</sup>

<sup>1</sup>LE STUDIUM Institute for Advanced Studies, 45000 Orléans, France

<sup>2</sup>School of Sciences, Yanshan University, Qinhuangdao, Hebei, 066004, PR China

<sup>3</sup>INSA Centre Val de Loire, Université d'Orléans, PRISME EA 4229, Bourges Cedex, 18022, France

## REPORT INFO

*Fellow:* Yiming Chen  
*From* Yanshan University, China  
*Host laboratory in region Centre-Val de Loire:* PRISME  
*Host scientist:* Driss Boutat  
*Period of residence in region Centre-Val de Loire:* January 2016 – December 2019

## Keywords :

*Fractional order systems, Numerical algorithms, Non-asymptotic algebraic observers, Approximation theory*

## ABSTRACT

During the past decades, fractional calculus has gained great interest and success in the field of automatic control. The research project was on numerical algorithms and observer design for fractional order systems. Various effective algorithms have been proposed to simulate different kinds of systems. Accurate and robust algebraic observers have been designed to estimate useful system information in noisy environment. An international conference has been held with the support of the project, which provided a platform for researchers to exchange results and advanced technology. Moreover, this project has provided an international collaboration opportunity for faculties and students both from INSA Centre Val de Loire and Yanshan University in China.

## 1- Introduction

In this project, the research professor fellow Yiming Chen comes from Yanshan University in China. He was invited to the region Centre for three months each year from 2016 to 2019. The host scientist Prof. Driss Boutat works at INSA Centre Val de Loire (CVL) in the laboratory of PRISME. Three researchers of INSA CVL were also involved in this project: Dr. Dayan Liu belonging to the PRISME Laboratory, Dr. Roger Serra and Dr. Serge Dos Santos belonging to the laboratories of LMR and U930 INSERM, respectively.

The objective of this project was to strengthen the research and teaching of the laboratories of PRISME, LMR, U930 INSERM and INSA CVL, as well as to create the link among these laboratories, INSA CVL and Yanshan University.

During this project, excellent research results on numerical algorithms and observer design for fractional order systems have been obtained and presented in 16 journal papers, an international

conference has successfully been held at Bourges, and a series of research talks and a series of courses have been provided by Prof. Chen. Moreover, a Memorandum of Understanding has been signed between INSA CVL and Yanshan University, which has permitted the exchange of faculties and students, especially 6 former master students from Yanshan University have got the opportunity to prepare a PhD thesis in INSA CVL.

Details of the previous achievements will be given in the following sections.

## 2- Research on fractional order systems

The birth of fractional calculus can be traced back to 1695 when Leibniz wrote a letter to L'Hospital, in which the concept of fractional derivative was mentioned for the first time, and many scholars have made great contributions to it since then. During the past decades, fractional calculus has gained great interest and success in many fields of science and engineering. For instance, fractional order systems (FOSs) and

Chen, Y.; Boutat, D.; Liu, D. Numerical Algorithms and Observer Design for Fractional Order Systems, *LE STUDIUM Multidisciplinary Journal*, 2019, 3, 44-48

<https://doi.org/10.34846/le-studium.141.04.fr.12-2019>

controllers have been applied to improve performance and robustness properties in control design.

On the one hand, when studying a FOS, an important task is to obtain the numerical output solution of FOSs from the input (simulation of FOSs). For this purpose, various remarkable and effective algorithms have been proposed in this project for different kinds of FOSs, such as fractional multi-order linear systems, fractional variable order linear systems, fractional order nonlinear systems, fractional order distributed parameter systems [4,5,6,9,10,16], by using different basis, such as wavelet functions, Bernstein polynomials, Legendre and Chebyshev orthogonal polynomials, etc.

On the other hand, in order to further deal with the system control analysis and synthesis, it is essential to obtain some information of FOSs, such as the integral and derivative of the output and the state. Indeed, the integral and derivative of the output are useful to design PID controllers. State information can help to understand the system dynamics, on which the designs of many control algorithms also depend. Due to the nonlocality of the fractional derivative operator, the real state of a fractional order system can be divided into two parts: the pseudo-state and an initialization function. However, for some applications, the knowledge of pseudo-state is enough to understand the behavior of a studied FOS. Since the previously mentioned information usually can't be measured by sensors, observers are needed to estimate them. For this purpose, two non-asymptotic algebraic observers have been designed in this project to estimate the fractional derivatives of the output and the pseudo-state of a class of fractional order linear systems in noisy environment, respectively [1,2]. These observers are accurate and robust against noises. Moreover, they can provide estimations within a finite time, which are useful for online applications. Moreover, two model-free fractional order differentiators have also been designed to estimate the fractional derivative of unknown signal [7,8].

### 3- Articles published in the framework of the fellowship

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Within the framework of the project, as LE STUDIUM RESEARCH PROFESSOR, Prof. Chen has published 15 articles with his students and the project partners Driss Boutat, Dayan Liu, Serge Dos Santos. Moreover, a paper is under review.

[1] Wei X., Liu D.Y., Boutat D., Chen Y.M., Algebraic fractional order differentiator based on the pseudo-state space representation, *Fractional Calculus and Applied Analysis*, Vol. 22, pp. 1395-1413, 2019.

[2] Wei Y.Q., Liu D.Y., Boutat D., Chen Y.M., An improved pseudo-state estimator for a class of commensurate fractional order linear systems based on fractional order modulating functions, *Systems & Control Letters*, Vol. 118, pp. 29-34, 2018.

[3] Wang L.F., Wu H.Q., Liu D.Y., Boutat D., Chen Y.M., Lur'e Postnikov Lyapunov functional technique to global Mittag-Leffler stability of fractional-order neural network with piecewise constant argument, *Neurocomputing*, Vol. 302, pp. 23-32, 2018.

[4] Wang L.P., Chen Y.M., Liu D.Y., Boutat D., Numerical algorithm to solve generalized fractional pantograph equations with variable coefficients based on shifted Chebyshev polynomials, *International Journal of Computer Mathematics*, Vol. 96, pp. 2487-2510, 2019.

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[8] Chen Y.M., Wei Y.Q., Liu D.Y., Boutat D., Chen X.K., Variable-order fractional numerical differentiation for noisy signals by wavelet denoising, *Journal of Computational Physics*, Vol. 11, pp. 338-347, 2016.

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[11] Wang L., Chen Y.M., Shifted-Chebyshev-polynomial-based numerical algorithm for fractional order polymer visco-elastic rotating beam, *Chaos, Solitons & Fractals*, vol. 132, pp. 109585, 2020.

[12] Wang Y.H., Chen Y.M., Shifted Legendre polynomials algorithm used for the dynamic analysis of viscoelastic pipes conveying fluid with variable fractional order model, *Applied Mathematical Modelling*, Vol. 81, pp. 159-176, 2020.

[13] Feng Y.J., Liu L.C., Chen Y.M., Numerical analysis of fractional-order variable section cantilever beams, *ICIC Express Letters*, Vol. 13, pp. 547-555, 2019.

[14] Yu C.X., Zhang J., Chen Y.M., Feng Y.J., Yang A.M., A numerical method for solving fractional-order viscoelastic Euler-Bernoulli beams, *Chaos, Solitons & Fractals*, Vol. 128, pp. 275-279, 2019.

[15] Wang Y.H., Chen Y.M., Dynamic analysis of the viscoelastic pipeline conveying fluid with an improved variable fractional order model based on shifted Legendre polynomials, *Fractal and Fractional*, Vol. 3, 2019.

[16] Wang L., Chen Y.M., Zhang X.J., Liu D.Y., Boutat D. and Wang Y.H., A numerical method with shifted Chebyshev polynomials for a set of

variable order fractional partial differential equations, submitted to *Numerical Methods for Partial Differential Equations* (under review).

#### 4- Conference organization

Thanks to the excellent research works of Prof. Chen, Prof. Boutat and Dr. Liu in the community of fractional calculus, they have successfully organized "2019 International Conference on Fractional Calculus Theory and Applications" (ICFCTA 2019) from April 25 to April 26 at Bourges. The conference was co-sponsored by LE STUDIUM Loire Valley Institute for Advanced Studies (France), National Institute of Applied Sciences (INSA) of Centre Val de Loire (France), Yanshan University (China), and the Technical Committee on Fractional Order Systems and Control of Chinese Association of Automation.

This conference provided a platform for researchers working on fractional calculus theory and applications to exchange results and advanced technologies. There were more than 40 participants from 11 countries attending the conference: Australia, China, France, Germany, Oman, Poland, Portugal, Saudi Arabia, Slovakia, Ukraine and USA. The conference contained 6 invited plenary lectures, 17 oral talks and 12 selected research papers for communication between participants. More details about this conference are available at:

<http://www.lestudium-ias.com/event/2019-international-conference-fractional-calculus-theory-and-applications-icfcta-2019>.

This news has been published in the journal of *Fractional Calculus and Applied Analysis*, 22(3), 2019. DOI: 10.1515/fca-2019-0032.

#### 5- Scientific oral communications

During the stay in both Campus of Bourges and Blois of INSA CVL, Prof. Chen has given the following research talks, as well as a series of courses on fractional calculus and differential equations for researchers and PhD students:

- Numerical solution study of fractional order and variable fractional order differential equations, THURSDAY Event organized by LE

STUDIUM, INSA CVL Campus of Bourges, May 5, 2016.

- Numerical methods based on wavelets and Bernstein polynomial with their applications, Seminar of PRISME Laboratory, Nouan le Fuzelier, Jun 27, 2016.

- Fractional and variable order fractional differential equation with their application, Seminar, INSA CVL Campus of Blois, May 12, 2017.

- Characterization of viscoelastic materials' mechanical properties and development of viscoelastic materials' mathematical model based on the theory of fractional calculus, Seminar, INSA CVL Campus of Blois, June 19, 2017.

- Numerical methods and their applications, 7 course of 3 hours, INSA CVL Campus of Blois, June 14-26, 2017.

#### **6- Cooperation between INSA CVL and Yanshan University**

Due to the stable collaboration between Prof. Chen and the project partners of INSA CVL, a Memorandum of Understanding (MoU) has been signed in November 2017 between INSA CVL and Yanshan University in order to develop the cooperation in scientific domains of mutual interest, such as exchange of faculties and students for research, internship and teaching, etc.

After the signature, Prof. Boutat and Dr. Liu were invited to visit Yanshan University from December 28, 2017 to January 2, 2018 by Prof. Chen. During the visit, Prof. Boutat and Dr. Liu have respectively given a talk for teachers and students of Yanshan University. Moreover, a seminar has been organized in the team of Prof. Chen for further discussions with his students.

Later, recommended by Prof. Chen, Dr. Liu has been nominated as Guest Professor by Yanshan University from 2018 to 2021. Thus, he is regularly invited to visit Yanshan University, and jointly supervises master students with Prof. Chen.

Moreover, in the framework of the MoU, two professors Haoran Liu and Yaqian Li have

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visited INSA CVL to talk about research programs and discuss potential cooperation projects from April 17 to 19, 2019. Besides the faculties' exchange, a student from INSA CVL has done an internship from April to August 2019 in Yanshan University.

#### **7- PhD Students training in INSA CVL**

From 2016, Prof. Chen has been recommending his master students from Yanshan University to prepare their PhD thesis in INSA CVL, financed by the China Scholarship Council (CSC). This is a good opportunity for INSA CVL to increase PhD candidates.

Prof. Boutat and Dr. Liu have got four PhD students. Yanqiao Wei has defended on 15 November 2019. Lifei Wang and Zhibo Wang are preparing their PhD thesis. In October 2020, Jie Liu will join the team of Prof. Boutat. Dr. Sera has also got a PhD student Xiaolin Li. Currently, another student Lei Wang is preparing to apply the high-level program of CSC to study in INSA CVL under the supervision of Prof. Boutat and Dr. Liu.

Moreover, during the stay in INSA CVL, Prof. Chen has also helped PhD students Xing Wei and Bainan Liu in the team of Prof. Boutat, and Yuzhu Wang in the team of Dr. Sera.

#### **8- Perspectives of future collaborations**

As shown in the proposal, this project was initially on observer design for distributed parameter systems and fractional order systems, and applications to nonlinear acoustics and viscoelastically damped structures.

During this project, most of the research works were focused on the numerical simulations of fractional order systems including fractional order distributed parameter systems. Two observers were designed for fractional order linear systems and two model-free fractional order differentiators were also developed. In the future, Prof. Chen and the project partners will continue their collaboration by working on observer design for fractional nonlinear systems and distributed parameter systems. The theory results obtained in this project make these perspectives faisable. Then, the results will be applied to nonlinear acoustics and

viscoelastically damped structures. Indeed, some preliminary works on fractional order modeling and numerical algorithms of viscoelastic materials have been obtained this year by Prof. Chen and his students [11,12,13,14,15].

In order to strengthen the built collaboration, Prof. Chen and the project partners will apply other European or Chinese research projects, continue to visit each other and to jointly supervise master students in Yanshan University to train future PhD candidates for INSA CVL.

Finally, based on the existing cooperation between INSA CVL and Yanshan University, more cooperations are expected.]

## **9- Conclusion**

Fractional calculus has a long history and has been becoming very useful in many scientific and engineering fields. Based on this research topic, this project was built and completed. Thanks to this project, researchers from INSA CVL and Yanshan University in China have established a solid collaboration by achieving a series of excellent research works published in international journals and by jointly supervising students. An international conference has also been organized with the project support, which provided an excellent opportunity for researchers and students in INSA CVL to exchange with international experts. Consequently, both research and teaching of the laboratories of PRISME, LMR, U930 INSERM and INSA CVL have been strengthened. Moreover, a link between INSA CVL and Yanshan University has been created, which has permitted exchange of faculties and students to each university. Finally, due to the results obtained in this project, more collaboration will be continued in the future.